What We Did Last Year

Calculus 12, Veritas Prep.

These problems should be fairly self-explanatory. Please write up your solutions/answers for the last four problems (8–12) on a separate sheet and hand them in.

1. Solve for x (i.e., find all values of x that make this equation true):

$$0 = 10\alpha x^{29} - 40g\alpha x^{27} - 5yx^2\alpha^2 + 20\alpha^2 gy$$

2. Sketch $y = \dfrac{x(x-5)(x+2)}{3(x+1)(x+2)}$

3. Sketch $f(x) = x^2(x+1)(x-20)(x+5)^3(2x+4)$

4. Evaluate: $\sin(\pi/3)$, $\cos(\pi/4)$, $\tan(5\pi/4)$, $\tan(5\pi/6)$.

5. Find $\frac{dy}{dx}$, given that $y = x^2 + x^{74} - \ln(x) - \log_3(x) + 51^x - e^x + \sin(x) - \cos(x)$

6. Find $\frac{dy}{dx}$, given that $y = x(x^2+2) - \sin(x^4 - x^{90}) + e^{\sin(x)} + \ln(\cos(x^2))$

7. Find $\frac{dy}{dx}$, given that $y = \dfrac{x^5 + x^{25}}{\sin(x)} + x^5\sin(x) + x^3\sin(x)e^{5x}$

8. A pig and a porcupine, connected by a spring, are perambulating away from a pine tree at perpendicular angles to each other. The pig is waddling at $5m/s$ and the porcupine is strolling at $3m/s$. After they've been fleeing the tree for twenty seconds, how quickly is the length of the spring changing? (Assume that, despite the spring force getting stronger and stronger, they keep constant velocities and trajectories. A much, much cooler version of this problem would have it entangled with Hooke's law, which is, after all, just a basic differential equation: force = -(some constant) · velocity, and since force = mass $\cdot\frac{dv}{dt}$ (where v is velocity), we have $m\frac{dv}{dt} = -kv$. I guess we'd need to know the masses of the animals. Anyway, I am totally excited about writing a problem like this sometime in the future.)

9. As the deputy director for operations of the Central Arizona Project, you are trying to design a new canal from Tempe to Tucson. The canal will be above ground, for some reason, and built using prefabricated sheets of an advanced polymer composite imported from Norway. The polymer sheets are one foot wide, and come in sections 39 feet long (to fit in standard shipping containers). Sensibly enough, you'll assemble them into a quasi-U shape, with one section as the bottom of the canal, and two sections as sides. Since you want to maximize the volume of delicious, life-providing water that this canal carries, what should the angle between the two side pieces and the bottom piece be? (At some point in the answer, the periodic trig identities may be helpful.)

10. Explain the significance of the equation $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$. Where does it come from?

11. What is a limit? (Write a paragraph.)

12. What is an antiderivative? Discuss.