$\sqrt{\textbf{Complex!}}$

Math 3

Some problems to do, right now, on a separate sheet of paper. Draw pictures for *all* of these problems.

7-1. Find the square roots of i. We've already done this—but we did it in *rectangular*. This time, do it in polar. You know what the answers ought to be, so make a geometric/polar argument for why those are the answers. (And do write the answers in polar form.) One of the roots is easy to find—how do we find the second one? Make some sort of geometric or polar argument for why the root in the lower-left quadrant is also a square root.

7-2. Remember how much you struggled to find the cube roots of i, and then the quartic roots? Do it again! Except this time, instead of multiplying out zillions of factors, solving doubly-quadratic equations, and losing plus-or-minus signs like a shell game on the boardwalk, do it in polar coordinates. One of the roots (for each set) is straightforward to find. How do you get the other ones? Draw all of them.

7-3. Was that more or less fun than finding the cubic and quartic roots of i in rectangular? (Answers may genuinely vary.)

7-4. Find all the septic roots of i. How many of them are there? How do you know there aren't more of them? Are you sure there aren't *eight* or even *nine* septic roots? Why, or why not? Note that "because Andrew/the fundamental theorem of algebra says so" isn't a good enough argument.

7-5. Find the cube roots of $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$. Draw them, too. How many of them are there? Are there really three of them? Are you sure there aren't four, or five, or six? How do you know?

7-6. Find all the cube roots of 12i.

7-7. Revisit our other favorite complex-numbers-model-organism: the sixth roots of 64! Find them in polar coordinates. Then find the sixth roots of 729 in polar (note that $729 = 3^6$). How do the answers differ?

7-8. Find all the quartic roots of 16. Then find all the quartic roots of 81. Like in the previous problem, how are these two sets of roots similar and different from each other?

7-9. Let's find some roots of 1! (The fancy math word for these are **roots of unity**, since "unity" is just a fancy word for "1".)

(a) Find all the cube roots of 1. Oh, you think there's only one? Find more of them. How many of them are there? Are you sure there aren't more? Draw them.

(b) Find all the quartic roots of 1. (How do these differ from the quartic roots of 16 and 81?)

(c) Find all the quintic roots of 1.

(d) Find all the ~~27th~~ vigintiseptic roots of 1.

(e) Find all the nth roots of 1 (i.e., come up with a formula!)

7-10. Find all the nth roots of x, for $x \in \mathbb{R}$. In other words, figure out a formula that will give you *all the roots* of *any real number*!!!

7-11. Suppose you have two complex numbers, both with the same angle, but with different radii. You take their nth roots. How do the two different sets of nth roots differ?

7-12. Suppose you have two complex numbers with the same radii but different angles. You take their nth roots. How do the two different sets of nth roots differ?

7-13. Let's find roots of some non-real non-imaginary complex numbers!

(a) Find all the square roots of the complex number given by the point on the unit circle with an angle of 30°.

(b) Find all the square roots of $\cos(\pi/4) + i\sin(\pi/4)$.

(c) Find all the square roots of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

(d) How are these square roots different from the square roots of 1?

(e) Now find the cube roots of all three of those points.

(f) Find the quartic roots of those three points.

(g) In general, how do we find the nth roots of some point on the unit circle?

7-14. Finally, find *every root* of *every complex number*! *One formula for everyyyyyyyything.*