Let's Multiply Things By i!

Math 3

Some problems to do, right now, on a separate sheet of paper:

2-1. Take the complex number 7. (Yes, it's a real number, but it's also a complex number.) Graph it on the complex plane. Then...

(a) Multiply it by i. What do you get? Graph it (on the same axes).

(b) Multiply *that* by i. (In other words, compute $(7 \cdot i) \cdot i$). Again, show it algebraically (i.e., as a mathy expression) and visually (i.e., graph it, on the same axes).

(c) Multiply it by i a third time, and again, show that result symbolically and visually.

(d) Do it a fourth time.

(e) Do it a fifth time.

(f) Do it a sixth time.

(g) What if you did it 793 times? What would the result be (both symbolically and visually)? Do you have to do all 792 antecedent calculations, or is there an easier way?

2-2. Now do the same thing(s) as in the previous problem, but with the following numbers:

(a) 5

(b) 1

(c) $6i$

(d) $3 + 2i$

(e) your favorite real number

(f) your favorite imaginary number

(g) your favorite non-real, non-imaginary complex number (i.e., a complex number that has both real and imaginary parts that aren't 0).

2-3. Now (and this is the **WHOLE POINT**), make a conjecture: what happens, *visually/geometrically*, when we multiply a number by i? (VERY IMPORTANT! ALL THOSE PREVIOUS QUESTIONS WERE LEADING UP TO THAT!!! THIS IS MAYBE THE MOST IMPORTANT THING WE WILL LEARN THIS MONTH!!!!!) (Maybe you already know it? Maybe you did it last year and remember?!? No matter!)

2-4. What about when we multiply a number by i^2? by i^3? by i^4? by i^5?

2-5. What about when we multiply a number by $i^{0.5}$? What about when we multiply a number by $i^{1.7}$? Guess!

2-6. Verify one of your guesses from the previous part. In class, we worked out that:

$$\sqrt{i} = i^{0.5} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

Take the number 7, and multiply it by this. Plot the result. Then multiply the result by $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ again, and plot it. Again! Again!

2-7. What about when we *divide* a number by i? If you're unsure about this, here are some things to try slash think about:

- Try repeating all the same subproblems as in the first problem, but with division instead of multiplication. What happens?
- Division and multiplication—how are they related?
- Suppose you have the complex number $7 \cdot i^3$. Where is it on the complex plane? If you divide off an i, to get $7i^2$, where is it? What if you divide off another i?

2-8. What if we multiply a number by a number that's not just a pure power of i?

(a) For example, what if we multiply 5 by $2i$? What do we get, symbolically and visually? How does the 2 change things from just multiplying it by pure i? What if we do that again? In other words, what is $5 \cdot (2i) \cdot 2i$, symbolically and visually? What if we multiply 5 by $2i$ three times? Four times? n times?

(b) Likewise, what if we multiply 5 by $3 + i$? What do we get, symbolically and visually? What if we multiply it by $3 + i$ twice (i.e., compute $5 \cdot (3 + i)(3 + i)$, and then plot it on the complex plane)? What about three times (i.e., $5 \cdot (3 + i)(3 + i)(3 + i)$? Four times? Five times? Do you really want to keep working that out? Why or why not? Is this frustrating? Is there a better way? Do you see a pattern?

(c) Here's the most general question: if we multiply the number $z_1 = a + bi$ by $z_2 = c + di$, what does that mean, visually and geometrically?