Two Problems
Calculus 11, Veritas Prep.

We know how to find the derivative of two functions multiplied together—that's what the product rule is for. What if I want to find the third derivative of two functions multiplied together? the fourth derivative? fifth? what if I want to find the nth derivative?

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d^2}{dx^2}[f(x)g(x)] = \quad ???$$

$$\frac{d^3}{dx^3}[f(x)g(x)] = \quad ???$$

$$\frac{d^4}{dx^4}[f(x)g(x)] = \quad ???$$

$$\vdots \qquad \vdots$$

$$\frac{d^n}{dx^n}[f(x)g(x)] = \quad ???$$

We know how to find the derivative of two functions multiplied together—that's what the product rule is for. But what if we have three functions multiplied together? what if we have four functions multiplied together? five? what if I have n functions multiplied together??? (For notational convenience as you try to figure this out, you may wish to write your functions as $f_1(x)$, $f_2(x)$, $f_3(x)$, and so on, because there are only 26 letters. And you might wish to drop the "(x)", too, since it creates a lot of redundant writing.)

$$\frac{d}{dx}[f_1 f_2] = f_1' f_2 + f_1 f_2'$$

$$\frac{d}{dx}[f_1 f_2 f_3] = \quad ???$$

$$\frac{d}{dx}[f_1 f_2 f_3 f_4] = \quad ???$$

$$\vdots \qquad \vdots$$

$$\frac{d}{dx}[f_1 f_2 f_3 \cdots f_n] = \quad ???$$